Steady-State Determination for RF Circuits Using Krylov-subspace Methods in SPICE

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Abstract — **A SPICE-based direct shooting-Newton method for the determination of the steady-state response of RF circuits has been developed. Different Krylov-subspace methods, including GMRES, CGS, BiCG, QMR, and BiCGSTAB, were used to solve the iterative equations generated by the shooting-Newton algorithm. The public-domain circuit simulator, SPICE, was used for the implementation of the new steady-state analysis. Compared to standard transient analysis for the determination of the steady-state response for non-linear circuits encountered in RF design, this new method is much more efficient. RF circuits that are difficult to simulate are evaluated. For larger circuits, the GMRES, QMR, and BiCGSTAB algorithms show the most improvement in the time to calculate the steady-state.**

I. Introduction

Given today's increasing demand for communication devices, including cellular telephones and other cordless devices, more effective methods are needed to examine integrated circuits that make up these devices in the steady-state mode. Quantities such as power, distortion, and noise are evaluated in the steady-state and need to be studied in detail [1]. Standard simulators, such as SPICE, can use transient analysis to determine the steady-state response by simulating until all the transients have died out. Unfortunately, for the type of circuits used in communication devices, this takes much too long for a detailed analysis to be made. Harmonic balance methods are not optimal for strongly non-linear circuits that we need to evaluate. Direct time-domain methods are a good choice for steady-state determination [2]-[3].

For SPICE-based circuit simulators, the time-domain shooting-Newton method is relatively easy to implement [4]-[5]. The direct shooting-Newton method of steady-state determination was proposed by Aprille and Tricke [2], and when implemented in an older version of SPICE [4], used the traditional Gaussian elimination to solve the iterate. Because of the computation costs, this limited the use of the algorithm to relatively small circuits. The newer Krylov-subspace methods can solve these equations generated by the steady-state response determination with much greater efficiency. The Krylov-subspace method of GMRES has been implemented with the direct shooting-Newton method [6]. However, it is not implemented in public-domain software, and does not include other Krylov-subspace methods for comparison.

There are two parts to this new method. First, the direct approach to steady-state determination is implemented in SPICE, second, several different Krylov-subspace methods, including BiCG and QMR are used to solve the iterate generated by the direct method [5]. Examples from microwave circuits are used to illustrate the new method.

II. Steady-State Determination

The shooting-Newton direct method of steady-state determination for circuits with periodic input was implemented in SPICE 3f5. In addition, the Krylov-subspace methods were also implemented in SPICE 3f5 for the solution of the iterate generated by that method. Direct methods, such as the shooting-Newton method presented here, were used to find the initial state needed to put the circuit directly in steady-state [2]. If the circuit equations are represented as the system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)\,, \qquad (1)$$

where **x** and **f** are n vectors, the vector **f** is periodic in time, t, and has a period of T. A constraint for achieving steady-state is that the transient effects have died off. This is represented by:

$$\mathbf{x}(0) = \mathbf{x}(T)\,. \qquad (2)$$

In other words, the solution at the end of the period is the same as the condition at the beginning of the period. This means that the circuit is in steady-state. The state transition function can be used to define the two-point boundary value problem, thus:

$$\mathbf{x}(0) - \phi(\mathbf{x}(t_0), t_0, T) = 0 \qquad (3)$$

where ϕ is the state transition function. The state transition function was implicitly derived; it was calculated at each timepoint until the end of the period. It is dependent on the initial state, x_0, the period of the response, T, and the starting time, t_0. We applied the shooting-Newton method to solve the boundary value problem that results in the following iteration:

$$\mathbf{x}_0(t)^{k+1} = \mathbf{x}_0(t)^k - \left[\mathbf{I} - \mathbf{J}_\phi\right]^{-1}\left[\mathbf{x}_0{}^k - \phi(\mathbf{x}(t_0), t_0, T)\right], \quad (4)$$

where k is the iteration index and $\mathbf{J}_\phi$ is the sensitivity matrix represented by:

$$\mathbf{J}_\phi = \frac{d}{dx}\phi(\mathbf{x}(t_0), t_0, T) \quad (5)$$

The sensitivity matrix was computed at the same time as the state transition function. Quantities needed for the calculation of the sensitivity matrix were already available at each timepoint from the transient analysis. The forming of the sensitivity matrix is computationally expensive. The iterate was solved and, using a user-defined limit, was considered converged. If not, the circuit was resimulated and another initial guess was used [2]-[4]. This process was continued until the steady-state was reached. The shooting-Newton method computed a set of capacitor voltages and inductor currents for the circuit so that if these voltages and currents are used as the initial conditions for the transient analysis, the circuit is directly in steady-state.

The accuracy of the direct steady-state determination and the accuracy of the Krylov-subspace methods are shown in Fig. 1.

The circuit simulated is a DC to 1 GHz class AB amplifier [7]. This circuit is difficult to simulate because the bias circuitry transients take a long time to settle out.

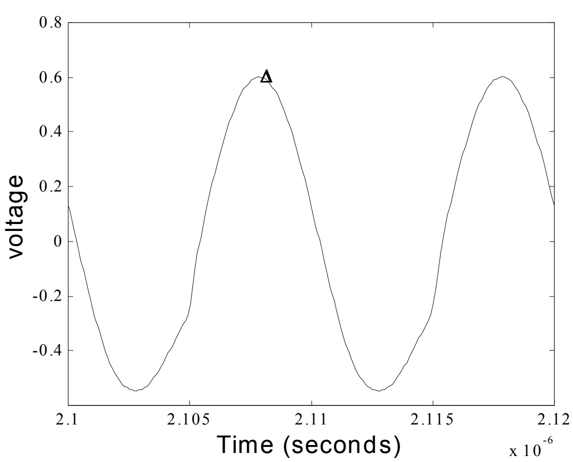

Fig. 1. Two periods of the transient analysis of the Class AB amplifier [7] in steady-state. The curves for all the methods lie on top of each other, as they should.

II. KRYLOV-SUBSPACE METHODS

The Krylov-subspace methods of GMRES, BiCG, QMR, CGS, and BiCGSTAB were incorporated into SPICE 3f5 for the solution of the linear system of equations generated by the shooting-Newton method to determine the steady-state response [8]. These methods are for general matrices, including the type that is generated by the shooting-Newton iteration (non-symmetric) [8]. In the following subsections, equation (4) was solved. The matrix being referred to is the sensitivity matrix $\mathbf{J}_\phi$.

The convergence figures were generated using a common-base Class C amplifier [9] operated at microwave frequencies. Appropriate changes were made to the original circuit for microwave operation. The circuit contained a total of 11 capacitors and inductors. The residual error was calculated within the Krylov-subspace method. The circuit was difficult to simulate using transient analysis because the biasing circuitry takes thousands of cycles for the transients to settle out.

A. GMRES

The Generalized Minimal Residual (GMRES) method generated a sequence of orthogonal vectors. The vectors were generated using a special method for Krylov-subspaces called the Arnoldi method. It used these vectors to do a least squares solution. One drawback of this method is that all the orthogonal vectors must be stored. So for large circuits, this storage need could be very large. It used the actual matrix and not its transpose for solution. The convergence behavior for the Class C amplifier is shown in Fig. 2.

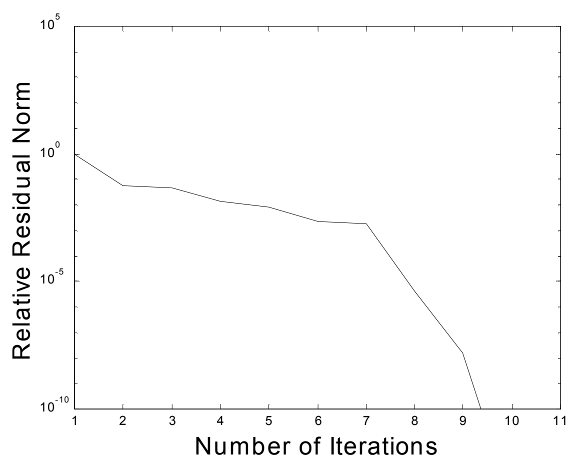

Fig. 2. Convergence behavior for Krylov-subspace method GMRES.

B. BiCG

BiCG is the Biconjugate Gradient method. It generated two sequences of vectors that are made mutually orthogonal to each other called bi-orthogonal. One of the sequences was generated by the original coefficient matrix, and the other by the transpose of that matrix. It used much less storage than GMRES, but had problems with convergence, and had two matrix-vectors products at each iteration. Fig. 3 shows its convergence behavior for the microwave example circuit.

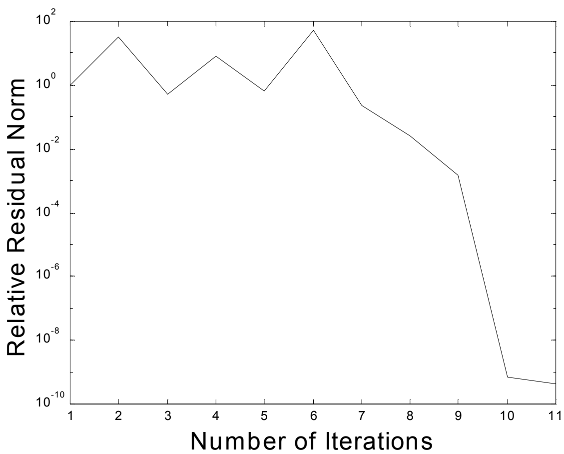

Fig. 3. Convergence behavior for Krylov-subspace method, BiCG.

C. QMR

The Quasi-Minimal Residual (QMR) method applied a least-squares solve and update to the BiCG residuals. QMR uses less storage than GMRES. It required matrix-vector multiplications of the original matrix and its transpose at each iteration step. Fig. 4 shows the convergence behavior for the QMR solution.

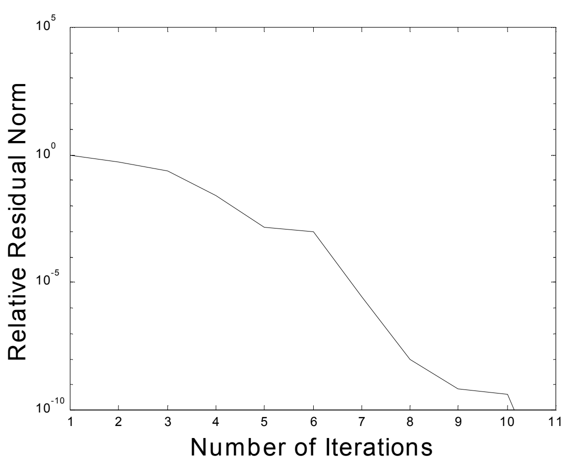

Fig. 4. Convergence behavior for the Krylov-subspace method QMR.

D. CGS

The Conjugate Gradient Squared (CGS) method is a variant of BiCG. It reformulated BiCG so that only the original matrix was needed and avoided transpose vector operations. Fig. 5 shows the convergence behavior for the BiCG solution.

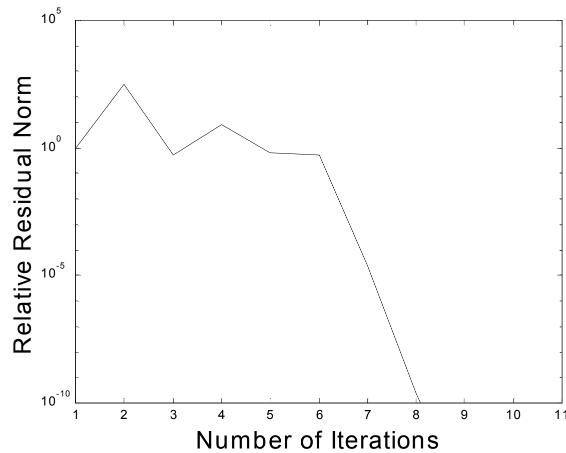

Fig. 5 Convergence behavior for Krylov-subspace method of CGS.

E. BiCGSTAB

Biconjugate Gradient Stabilized (BiCGSTAB) method is a variant of BiCG that used different updates to avoid using the transpose of the matrix. The convergence behavior for the example is shown in Fig. 6.

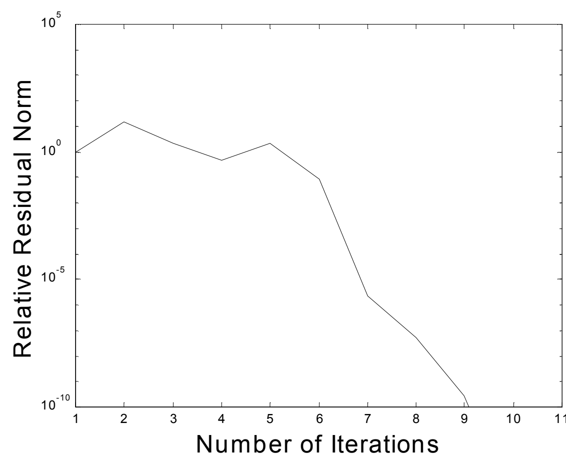

Fig. 6 Convergence behavior for the Krylov-subspace method BiCGSTAB.

III. RESULTS

In SPICE 3f5, the direct steady-state determination using the shooting-Newton method resulted in significant savings in computational time and resources (Table 1). The shooting-Newton method is also an accurate way to

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determine the steady-state response (Fig. 1). The system is in steady-state when the error is less than the user-specified value. The error measured was obtained using the maximum difference between the state of the circuit after a cycle of transient analysis and the initial state used for that cycle. Even greater efficiency was found using the Krylov-subspace methods for solution of the iterate generated by this method (Table 1).

Table 1. Summary of Krylov-subspace methods and transient analysis results of steady-state determination for a Class C amplifier.

Method Used for Steady-State Determination	Time (Seconds) To Reach Steady-State	Shooting-Newton Iterations	Transient Analysis Cycles
Transient analysis	67.8	-	>3000
Gaussian Elimination	1.76	26	35
GMRES	1.53	26	35
CGS	1.68	26	35
BiCG	1.79	26	35
BiCGSTAB	1.53	26	35
QMR	1.53	26	35

The convergence behavior (Fig. 2 through Fig. 6) for the solution of the iterate, generated by the shooting-Newton, pointed out the difficulty in convergence for some the Krylov-subspace methods. For some methods, such as CGS and BiCG, the residual is not reduced at each step of the iteration. The GMRES, QMR and BiCGSTAB algorithms converged the fastest. CGS also had fast convergence, but its convergence was very irregular.

V. Conclusion

The shooting-Newton direct steady-state determination was implemented in SPICE 3f5. The iterate generated by the method can be solved by using standard Gaussian elimination, or the different Krylov-subspace methods of GMRES, BiCG, QMR, CGS, and BiCGSTAB.

The direct method of steady-state determination is shown to be efficient and accurate (Fig. 1) when analyzing RF circuits that have difficulty in the standard transient analysis available in SPICE 3f5 (Table 1). This efficiency means shorter simulation times and less computer storage needs.

The Krylov-subspace methods make for even greater computer efficiency (Table 1). All the Krylov methods are much faster than the transient analysis approach of simulation until all the transients have died out. The Krylov-subspace methods give mixed results in terms of efficiency (Table 1) over the standard solution method of Gaussian elimination. The GMRES, QMR and BiCGSTAB algorithms show the fastest convergence to solution. Analysis with larger RF circuits is needed in order to investigate whether one Krylov-subspace method would be more computationally efficient in terms of storage and operation count for RF circuits.

The direct steady-state determination using Krylov-subspace methods was shown to be an efficient and accurate method. Its application to larger RF circuits should result in even greater computer resource savings.

Acknowledgement

The authors wish to acknowledge the assistance of Kenneth Kundert at Cadence Design for the microwave example circuit used for analysis, and Misha Kilmer of Tufts University for many discussions on Krylov methods.

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